Laser Physics I (PHYC/ECE 464)

Homework #4, Due Monday, Sept. 26, Fall 2022

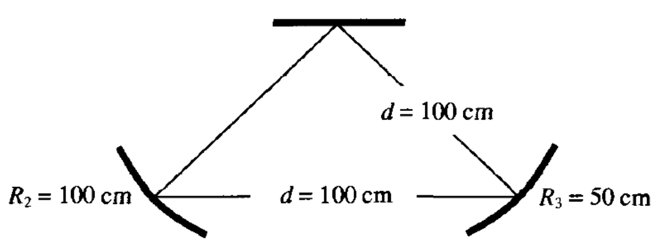
1 Consider a linear combination of two equal amplitude $\mathrm{TEM}_{m,p}$ modes given by:

$$\mathbf{E} = E_0 \left\{ (\mathrm{TEM}_{1,0})\mathbf{a}_y \pm j(\mathrm{TEM}_{0,1})\mathbf{a}_x \right\}$$

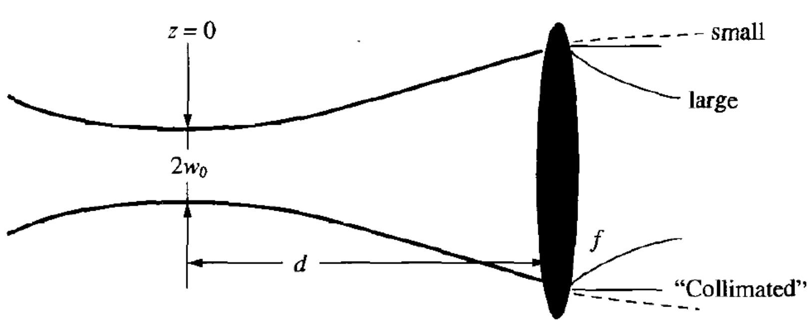
(a) Sketch the "dot" pattern or equal intensity contours for each component (i.e., $\mathbf{a}_x$ or $\mathbf{a}_y$). Indicate the direction of the electric field.
(b) Sketch the pattern for the linear combination.
(c) Label the positions where the intensity is a maximum and a minimum. (This is sometimes referred to as the "donut mode" or $\mathrm{TEM}^*_{0,1}$.

2 Is the cavity shown below stable? Demonstrate the logic of your answer by (a) constructing a unit cell starting at the flat mirror, (b) finding the $ABCD$ matrix for that cell, and (c) applying the stability criteria. (d) What are the circumstances under which the quantity $[AD - BC]$ can be different from 1? Why is $AD - BC$ always equal to 1 for a cavity? (Ignore astigmatism)

Bonus: Show the effect of astigmatism on the stability considering that the cavity axis below forms an equilateral triangle.



3 A focused Gaussian beam reaches its minimum spot size w_0 at $z = 0$ where $R = \infty$ and then propagates to a thin lens of focal length f located a distance d from $z = 0$. If w_0 is large, then the beam exiting the lens will be focused. If it is too small, then the lens merely reduces the far field spreading angle. Find the critical value of w_0 such that the output beam is "collimated"; that is, $R(z = d^+) = \infty$ also.



4. Using Eq. (5.2.8), write z_0^2 in terms of the g-paramters (i.e. g_1 and g_2). From this, derive the stability condition of the cavity in terms of g_1 and g_2. Compare this with the geometric optics results obtained eralier.